Delaunay Triangulation and Its Applications

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Abstract **- Data collection, data retention and analysis are becoming more and more important every day because of the fact that technology is involved almost all our life. The processing and analysis of the data can become more difficult with increasing precision.**

In three-dimensional surface modeling, due to the increase in sensitivity and data size depending on the surface state and extent of the surface, collecting and processing the data may become difficult. As a solution to this situation, we can see that Computational Geometry is used extensively. Computational Geometry derives intermediate interpolations by taking the start and end data as references instead of keeping each data separately. In this way, it is possible to model by determining intermediate values based on the mentioned reference points.

There are various methods in Computational Geometry such as intersection detection, point position and triangulation. According to the needs, a solution way can be produced by various geometric computations. In our study, "Delaunay Triangulation" which is the most used type of triangulation methods will be examined.

Keywords **- Delaunay Triangulation, Surface Modeling, Computational Geometry.**

I. INTRODUCTION

THE importance of developing technology in human life is increasing day by day. Every day, technology is spreading to more areas. Existing technologies are further developed and, accordingly, the need for data acquisition and data processing is also increasing. New needs arise, such as larger capacity memory units, more powerful processors, faster data transmission.

Small size items that we can use in our home can be made with 3D printers. Robot technology can now be designed to be close to reality. Historic buildings, museums, uninhabited cities and many other places can be seen and traveled through a computer program while sitting at home. 3D applications are developed with technology make our lives easier. In addition, technological requirements are increasing.

In some cases, these requirements can be met by installing systems that are much stronger than normal, but these needs may become unobtainable as the scope of the data expands. In these cases, generalization of the data may be the solution. This method facilitates the processing and management of the data.

II. MODELING METHODS

Surface modeling currently used in 3D applications is facilitated in terms of visualization, but it is still not possible to obtain at the desired level, store and analyze data in a topological sense. Although it is possible to model smooth surfaces or periodic rough surfaces, the situation is different especially on irregularly distributed surfaces.

When we think of a three-dimensional irregular surface, there are many complicated relationships between many parameters for modeling this area on a computer. One of the most common problems in cartography is surface digitization in order to work on irregular surfaces. It is possible only to digitize a map surface for a certain area and model it in a computer environment, although this is not possible exactly to everywhere. Usually the area, region, island, parcel etc. each area is estimated within itself by making measurements on the basis. Relation with other regions and areas is not determined. In this case, measurements are made in the relevant region, reference points are determined and intermediate values such as position and height according to these reference points are determined by interpolation method. In this way, a whole surface modeling is done.

Since it is not possible to model all real world surfaces in a one-to-one manner, the digitized model, which can be regarded as the primary model, is obtained by primarily structuring and generalizing spatial data. In the model generalization, it is the basic aim to prepare the data in the density and structure in accordance with the secondary models to be presented to the user by visualization and to obtain the cartographic model [1].

Various methods are used to generalize topological data so that it can be compiled as much as possible. Various geometric calculations can be made that vary according to the structure of the data to be processed, the requirements of the analyzes to be performed, and the data to be obtained. Here are some of them [2,3]:

a. Geometric Search
b. Concave Wall Creation
c. Separation by triangles
d. Finding proximity between objects
e. Finding the intersection

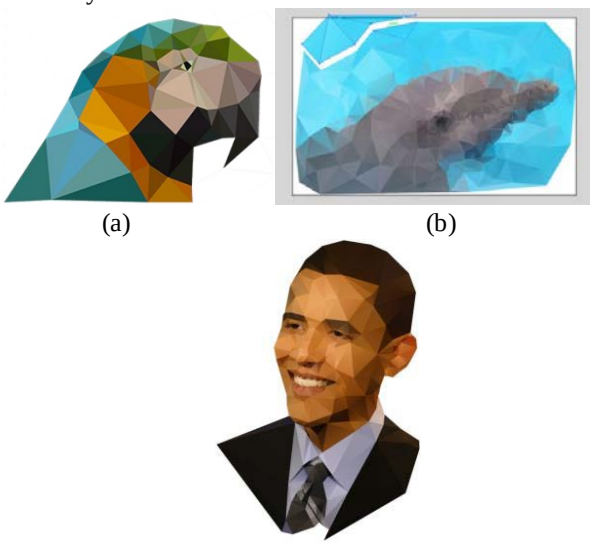
Within these methods, "Separation by triangles" method is frequently used within the scope of geographical information systems. Among the triangulation methods, "Delaunay triangulation" is the most commonly used triangulation method in surface modeling.

III. DELAUNAY TRIANGULATION

The main method used to model the surfaces is the "Delaunay Triangulation" method. In Turkish it is known as "Delaunay Üçgenlemesi". This method developed by Boris Delaunay in

1934.

The Delaunay triangulation relies on the reference points on a 2D or 3D surface, linking these points. It is a method that provides modeling by generating intermediate values by interpolation method. It is also used in different areas. However, map engineers seem to be used extensively in the models they need.



(a) (b)

(c)

Figure 1: Delaunay Triangulation applied pictures

We can list the important features of the Delaunay Triangulation as follows.

- It makes sense. It is definite and only one result.
- Independent from initial reference point.
- The resulting triangles are as equilateral triangles as possible. Due to the formation of very narrow angled triangles, a linear connection between the distant and non-directly related points is prevented [4].
- There is no other empty spot in the periphery of the diagram.
- The convex frame of the dataset is contained within the cluster. The convex frame of a point set is the smallest polygon that takes in that point set.
- The circle of reference points is located in the right triangle formed by the pair of points closest to each other.
- It forms the edge of a triangle with the right piece, which connects each point with another point closest to itself [5].

Delaunay Triangulation occupies a very important place in computational geometry [6]. It is necessary to define the Voronoi diagram in order to understand this triangular model which is so important.

IV. VORONOI DIAGRAM

The Voronoi diagram is also referred to in the literature as the Dirichlet, Thiessen, or Wigner-Seithz diagrams. Any point of the set of endpoints located in the plane is called the "Voronoi polygon (Field)" instead of the geometric plane of the points located closer to the other points in the cluster. The union of the Voronoi polygons of all points in the cluster forms the "Voronoi Diagram" of that cluster [2].

In the Voronoi diagram, the edges of the Voronoi polygon from the lines separating the points from the areas of the other points separate the point inside the area from the other neighboring points. The lines joining the reference points form the middle struts of the edges of the Voronoi Polygon.

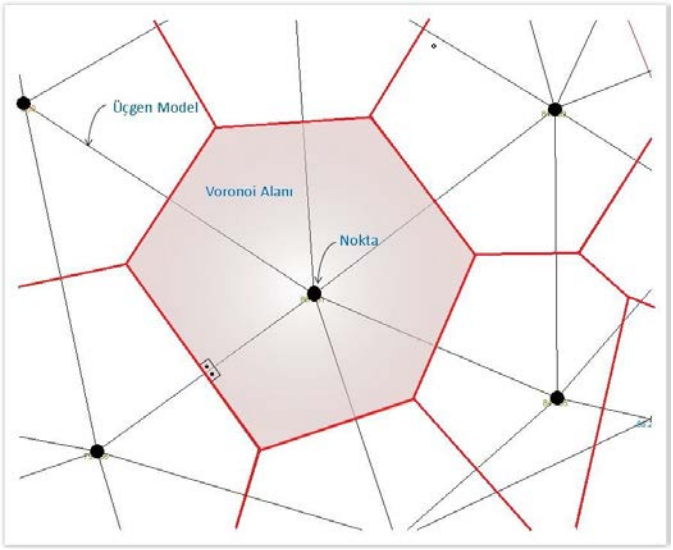
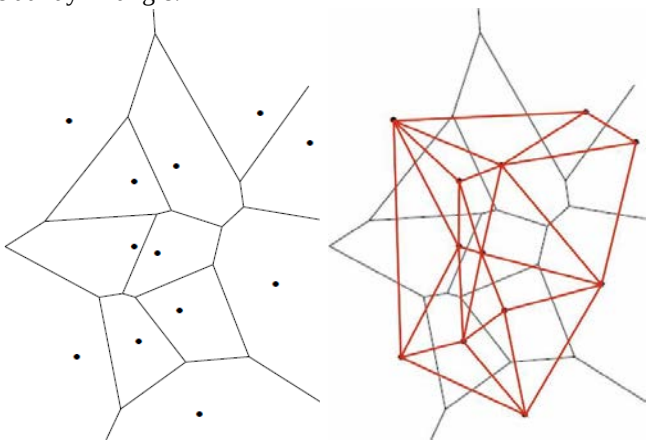


Figure 2: Voronoi polygon [7]

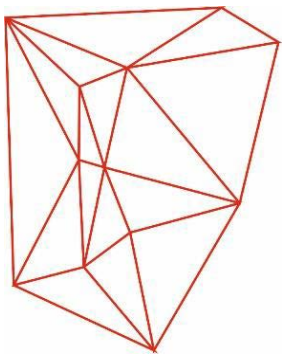
A. *Voronoi Polygon*

The Voronoi Polygon belonging to one point distinguishes that point from the other points adjacent to it. The other points closest to a point are defined as the points adjacent to that point. The middle struts that pass through the midpoints of the lines connecting the point and neighboring points are forming the edges of the polygon [7].

After the reference points are determined, the Voronoi diagram can be extracted. Voronoi diagram is a definite structure used for nearest point problems. In the Voronoi diagram, the Delaunay triangulation can be obtained as a result of connecting the reference points with each other reference points in their neighboring areas. In other words, the Voronoi diagram formed by the Voronoi polygons forms the basis of the Delaunay Triangle.



(a) (b)



(c)

Figure 3: Obtained samples of Delaunay triangle from Voronoi diagram

First of all, Voronoi Polygons are obtained by taking the midpoint sheds of the determined reference points. Then the triangulation is completed by obtaining the connections by the Delaunay triangulation method of these reference points.

V. The Importance Of Delaunay Triangulation In Geographical Information Systems

The modeling of generalization in Geographical Information Systems is a complex task because it depends on a wide variety of parameters. In addition to the roads, it is required to perform network analysis with parameters such as water network, sewer line, electricity lines. Generalization methods are gaining importance in order to make all these structures together and to make analysis such as shortest path analysis on the whole.

Bildirici and Selvi, in their studies on the generalization methods used in the Geographic Information Systems, the importance of triangulating and analyzing the generalization methods used in Geographical Information Systems with their strengths and weaknesses;

"In their work, four methods that have been assessed as feasible from a large number of previously published methods have been analyzed with their strengths and weaknesses. As a result of the study, it is planned to apply the triangulation method which is evaluated as the best in terms of applicability with macro support in a GIS software in the near future in order to create path axes. As a result, the following suggestions were made:

"It is for these reasons that field-line transformations are needed to be automated. As a result of the study, it is planned to apply the triangulation method which is evaluated as the best in terms of applicability with macro support in a GIS software in the near future in order to create path axes "[8].

Delaunay Triangulation, which has an important place in analysis methods in geographical information systems, is a preferred method of triangulation in terms of definite and single outcome.

VI. Conclusion

The size of the data to be analyzed is the factors that influence the application of the method of factors such as the number of reference points. It is important that a stable algorithm is used for the calculation-based software to work properly. The fact that Delaunay triangulation is the only result can be seen as the reason why the preference is high in this respect. However, it cannot be expected that the software will be successful in practice alone. Although the equipment has an important place here, it is expected that it may be insufficient depending on the size of the data to be processed.

Since the earth has an irregular surface, data entry at every point may be correct in terms of the health of the measurement, but it is not possible to apply it in practice. It is possible to perform modeling by making measurements at more points and producing intermediate interpolations as much as possible. When the applications are examined, independent measurements are made and models are made. Depending on the variation of the data processing capacity according to the number of points, the speed of the software can be seen as a disadvantage in terms of achieving healthy results.

In addition to this disadvantage, the more data entry is made, the higher the modeling accuracy. But besides this, data can be a serious problem in storage. If the same area is considered to have more than one network, a large-area modeling of all these networks, such as surface, road, waterline, electricity network, will require high capacity storage space. Apart from this, the data processing speed will also decrease accordingly. In addition to speed, the capacity problem can be considered as a disadvantage.

Although these are disadvantages, Yanalak M. in his work on topological surface modeling suggests that "Delaunay triangulation is the only meaning and that triangles are the closest triangles to the triangles, so it would be a good choice to use these triangles as a base in studies to be done on the network" [2,3].

As a result, "Delaunay Triangulation" alone is not enough. However, within the topological surface modeling methods, we can say that it is the most suitable modeling method in terms of applicability and efficiency.

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